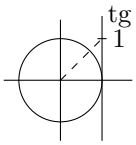
Určitý integrál = Horní mez − Spodní mez

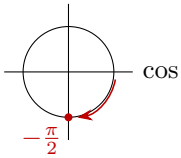
$$\int_{-2}^{1} \frac{\mathrm{d}t}{t^2+1} = \Big[\operatorname{arctg} t\Big]_{-2}^{1} = \operatorname{arctg} 1 - \operatorname{arctg}(-2) = \boxed{\frac{\pi}{4} - \operatorname{arctg}(-2)}$$



$$\int_{-1}^{1} \left(2x^3+3x^2+5\right)\,\mathrm{d}x = \left[2\cdot\frac{x^4}{4}+3\frac{x^3}{3}+5x\right]_{-1}^{1} = \frac{1^4}{2}+1^3+5\cdot 1-\left(\frac{(-1)^4}{2}+(-1)^3+5\cdot(-1)\right) =$$
$$= \frac{1}{2}+1+5-\left(\frac{1}{2}-1-5\right) = \frac{1}{2}+6-\left(\frac{1}{2}-6\right) = \frac{1}{2}+6-\frac{1}{2}+6 = \boxed{12}$$

$$\int_0^1 \frac{x-1}{x+2}\,\mathrm{d}x = \int_0^1 \frac{x+2-2-1}{x+2}\,\mathrm{d}x = \int_0^1 \frac{x+2-3}{x+2}\,\mathrm{d}x = \int_0^1 \underbrace{\frac{x+2}{x+2}}_{=1} - 3\frac{1}{x+2}\,\mathrm{d}x =$$
$$= \Big[x-3\cdot\ln|x+2|\Big]_0^1 = 1-3\cdot\ln|1+2|-(0-3\ln|0+2|) =$$
$$= 1-3\ln 3+3\ln 2 = 1-\ln 3^3+\ln 2^3 = 1+\ln 8-\ln 27 =$$
$$= \boxed{1+\ln\frac{8}{27}}$$

$$\int_{-\frac{\pi}{2}}^{\pi} (x-1)\sin x\,\mathrm{d}x = \left|\begin{array}{ll} u = x-1 & u' = 1 \\ v' = \sin x & v = -\cos x \end{array}\right| = \Big[(x-1)\cdot(-\cos x)\Big]_{-\frac{\pi}{2}}^{\pi} - \int_{-\frac{\pi}{2}}^{\pi} (-\cos x)\,\mathrm{d}x =$$
$$= \left[(\pi-1)\cdot(-\cos\pi) - \left[\left(-\frac{\pi}{2}-1\right)\cdot\underbrace{\left(-\cos\left(-\tfrac{\pi}{2}\right)\right)}_{=0}\right]\right] + \Big[\sin x\Big]_{-\frac{\pi}{2}}^{\pi} =$$



$$= (\pi-1)\cdot\underbrace{\left(-(-1)\right)}_{+} + \underbrace{\left[\sin\pi - \sin\left(-\tfrac{\pi}{2}\right)\right]}_{=0} =$$



$$= \pi - 1 + \left(-(-1)\right) = \pi - 1 + 1 = \boxed{\pi}$$

$$\int_{-1}^{1}(5x+2)\,e^{-2x}\,\mathrm{d}x = \left|\begin{array}{ll} u=5x+2 & u'=5 \\ v'=e^{-2x} & v \overset{*}{=} -\frac{1}{2}e^{-2x} \end{array}\right| =$$

$$= \left[(5x+2)\cdot\left(-\tfrac{1}{2}e^{-2x}\right)\right]_{-1}^{1} - \int_{-1}^{1} 5\left(-\tfrac{1}{2}e^{-2x}\right)\mathrm{d}x =$$

$$= (5\cdot 1+2)\cdot\left(-\tfrac{1}{2}e^{-2}\right) - \left((5\cdot(-1)+2)\cdot\left(-\tfrac{1}{2}e^{2}\right)\right) + \frac{5}{2}\int_{-1}^{1} e^{-2x}\,\mathrm{d}x =$$

$$= -\frac{7}{2}e^{-2} - \left(+\frac{3}{2}e^{2}\right) + \frac{5}{2}\left[-\frac{1}{2}e^{-2x}\right]_{-1}^{1} =$$

$$= -\frac{7}{2}e^{-2} - \frac{3}{2}e^{2} + \frac{5}{2}\left(-\frac{1}{2}e^{-2} - \left(-\frac{1}{2}e^{+2}\right)\right) = -\frac{7}{2}e^{-2} - \frac{3}{2}e^{2} - \frac{5}{4}e^{-2} + \frac{5}{4}e^{2} =$$

$$= \boxed{-\frac{19}{4}e^{-2} - \frac{1}{4}e^{2}}$$

Pomocný výpočet:

$$^{*}\int e^{-2x}\,\mathrm{d}x = \left|\begin{array}{l} t=-2x \\ \mathrm{d}t=-2\,\mathrm{d}x \\ \frac{\mathrm{d}t}{-2}=\mathrm{d}x \end{array}\right| = \int e^{t}\,\frac{\mathrm{d}t}{-2} = -\frac{1}{2}e^{t} = \boxed{-\frac{1}{2}e^{-2x}}$$

$$\int_{0}^{2}\frac{x}{\sqrt{8-x^2}}\,\mathrm{d}x = \left|\begin{array}{l} 8-x^2=t \\ -2x\,\mathrm{d}x=\mathrm{d}t \\ \mathrm{d}x=\frac{\mathrm{d}t}{-2x} \end{array}\right| = \int_{8}^{+4}\frac{\cancel{x}}{\sqrt{t}}\,\frac{\mathrm{d}t}{\cancel{-2x}} =$$

MUSÍ se OTOČIT MEZE!

$$x=2 \;\longrightarrow\; t=8-2^2=+4$$
$$x=0 \;\longrightarrow\; t=8-0^2=8$$

$$= +\frac{1}{2}\int_{4}^{8} t^{-\frac{1}{2}}\,\mathrm{d}t = \frac{1}{2}\left[\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right]_{4}^{8} = \frac{1}{2}\left[2\sqrt{t}\right]_{4}^{8} = \frac{1}{2}\left[2\sqrt{8}-2\sqrt{4}\right] =$$

$$= \frac{1}{2}\left(2\sqrt{8}-4\right) = \sqrt{8}-2 = \boxed{2\sqrt{2}-2}$$

$$\int_{0}^{2}\ln(1+x)\,\mathrm{d}x = \left|\begin{array}{l} 1+x=t \\ \mathrm{d}x=\mathrm{d}t \end{array}\right| = \int_{1}^{3} 1\cdot\ln t\,\mathrm{d}t = \left|\begin{array}{ll} u=\ln t & u'=\frac{1}{t} \\ v'=1 & v=t \end{array}\right| =$$

$$x=0 \;\to\; t=1+0=1$$
$$x=2 \;\to\; t=1+2=3$$

$$= \left[t\cdot\ln t\right]_{1}^{3} - \int_{1}^{3}\cancel{t}\cdot\frac{1}{\cancel{t}}\,\mathrm{d}t = \left[3\ln 3 - 1\ln 1\right] - \left[t\right]_{1}^{3} =$$

$$= 3\cdot\ln 3 - 0 - (3-1) = \boxed{3\cdot\ln 3 - 2}$$