Soustava, matice – řešení

$$\begin{array}{rcrcrcrcrcl} 3x_1 & - & 2x_2 & + & 6x_3 & + & 2x_4 & - & 4x_5 & = & 5 \\ x_1 & & & + & 2x_3 & - & x_4 & + & 2x_5 & = & 3 \\ x_1 & + & 2x_2 & + & 2x_3 & & & & & = & 1 \\ 2x_1 & - & 6x_2 & + & 4x_3 & + & 2x_4 & - & 4x_5 & = & 5 \end{array}$$

$$\left(\begin{array}{ccccc|c} 3 & -2 & 6 & 2 & -4 & 5 \\ 1 & 0 & 2 & -1 & 2 & 3 \\ 1 & 2 & 2 & 0 & 0 & 1 \\ 2 & -6 & 4 & 2 & -4 & 5 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 0 & 2 & -1 & 2 & 3 \\ 3 & -2 & 6 & 2 & -4 & 5 \\ 1 & 2 & 2 & 0 & 0 & 1 \\ 2 & -6 & 4 & 2 & -4 & 5 \end{array}\right) \begin{array}{l} \\ \cdot(-3) \\ \cdot(-1) \\ \cdot(-2) \end{array}$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 0 & 2 & -1 & 2 & 3 \\ 0 & -2 & 0 & 5 & -10 & -4 \\ 0 & 2 & 0 & 1 & -2 & -2 \\ 0 & -6 & 0 & 4 & -8 & -1 \end{array}\right) \begin{array}{l} \\ + \\ \cdot(-3) \end{array} \sim \left(\begin{array}{ccccc|c} 1 & 0 & 2 & -1 & 2 & 3 \\ 0 & -2 & 0 & 5 & -10 & -4 \\ 0 & 0 & 0 & 6 & -12 & -6 \\ 0 & 0 & 0 & -11 & 22 & 11 \end{array}\right) \begin{array}{l} \\ \\ \cdot\frac{1}{6} \\ \cdot\frac{1}{11} \end{array} \sim$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 0 & 2 & -1 & 2 & 3 \\ 0 & -2 & 0 & 5 & -10 & -4 \\ 0 & 0 & 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & -1 & 2 & 1 \end{array}\right) + \sim \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \left(\begin{array}{ccccc|c} 1 & 0 & 2 & -1 & 2 & 3 \\ 0 & -2 & 0 & 5 & -10 & -4 \\ 0 & 0 & 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \end{array} \begin{array}{l} (***) \\ (**) \\ (*) \end{array}$$

$h(A) = 3, \quad h(A_r) = 3, \quad n = 5$	FROBENIOVA VĚTA
$h(A) = h(A_r) \to$ má řešení	
$h(A) < n \to \infty$ řešení, $\quad n - h(A) = 5 - 3 = 2$ parametry	

$\underline{x_5 = t}$

$$\begin{aligned} (*) \to \; & x_4 - 2x_5 = -1 \;\Rightarrow\; \underline{x_4 = 2x_5 - 1 = 2t - 1} \\ (**) \to \; & -2x_2 + 5x_4 - 10x_5 = -4 \\ & -2x_2 + 5(2t-1) - 10t = -4 \\ & -2x_2 + 10t - 5 - 10t = -4 \\ & -2x_2 = -4 + 5 = 1 \;\Rightarrow\; \underline{x_2 = -\tfrac{1}{2}} \\ (***) \to \; & x_1 + 0x_2 + 2x_3 - x_4 + 2x_5 = 3 \\ & x_1 + 2x_3 - (2t-1) + 2t = 3 \\ & x_1 + 2x_3 - 2t + 1 + 2t = 3 \\ & x_1 + 2x_3 = 3 - 1 = 2 \;\Rightarrow\; \underline{x_3 = u} \\ & x_1 = 2 - 2x_3 = \underline{2 - 2u} \end{aligned}$$

$$x_1 = 2 - 2u, \quad x_2 = -\tfrac{1}{2}, \quad x_3 = u, \quad x_4 = 2t - 1, \quad x_5 = t, \quad u, t \in \mathbb{R}$$

$$A \cdot B = \begin{pmatrix} 1 & 2 & 4 \\ -1 & 0 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 8 \\ 5 & 4 & 6 \\ 7 & 9 & 3 \end{pmatrix} =$$

$$= \begin{pmatrix} 1\cdot 1 + 2\cdot 5 + 4\cdot 7 & 1\cdot 2 + 2\cdot 4 + 4\cdot 9 & 1\cdot 8 + 2\cdot 6 + 4\cdot 3 \\ -1\cdot 1 + 0\cdot 5 + 5\cdot 7 & -1\cdot 2 + 0\cdot 4 + 5\cdot 9 & -1\cdot 8 + 0\cdot 6 + 5\cdot 3 \end{pmatrix} = \boxed{\begin{pmatrix} 39 & 46 & 32 \\ 34 & 43 & 7 \end{pmatrix}}$$

$\underset{3,3}{B} \cdot \underset{2,3}{A} \rightarrow$ nelze násobit

$$A \cdot C = \begin{pmatrix} 1 & 2 & 4 \\ -1 & 0 & 5 \end{pmatrix} \cdot \begin{pmatrix} 5 & 1 \\ -1 & 0 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1\cdot 5 + 2\cdot(-1) + 4\cdot 2 & 1\cdot 1 + 2\cdot 0 + 4\cdot 3 \\ -1\cdot 5 + 0\cdot(-1) + 5\cdot 2 & -1\cdot 1 + 0\cdot 0 + 5\cdot 3 \end{pmatrix} = \boxed{\begin{pmatrix} 11 & 13 \\ 5 & 14 \end{pmatrix}}$$

$$C \cdot A = \begin{pmatrix} 5 & 1 \\ -1 & 0 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 4 \\ -1 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 5\cdot 1 + 1\cdot(-1) & 5\cdot 2 + 1\cdot 0 & 5\cdot 4 + 1\cdot 5 \\ -1\cdot 1 + 0\cdot(-1) & -1\cdot 2 + 0\cdot 0 & -1\cdot 4 + 0\cdot 5 \\ 2\cdot 1 + 3\cdot(-1) & 2\cdot 2 + 3\cdot 0 & 2\cdot 4 + 3\cdot 5 \end{pmatrix} = \boxed{\begin{pmatrix} 4 & 10 & 25 \\ -1 & -2 & -4 \\ -1 & 4 & 23 \end{pmatrix}}$$

$$B \cdot C = \begin{pmatrix} 1 & 2 & 8 \\ 5 & 4 & 6 \\ 7 & 9 & 3 \end{pmatrix} \cdot \begin{pmatrix} 5 & 1 \\ -1 & 0 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1\cdot 5 + 2\cdot(-1) + 8\cdot 2 & 1\cdot 1 + 2\cdot 0 + 8\cdot 3 \\ 5\cdot 5 + 4\cdot(-1) + 6\cdot 2 & 5\cdot 1 + 4\cdot 0 + 6\cdot 3 \\ 7\cdot 5 + 9\cdot(-1) + 3\cdot 2 & 7\cdot 1 + 9\cdot 0 + 3\cdot 3 \end{pmatrix} = \boxed{\begin{pmatrix} 19 & 25 \\ 33 & 23 \\ 32 & 16 \end{pmatrix}}$$

$\underset{3,2}{C} \cdot \underset{3,3}{B} \rightarrow$ nelze násobit