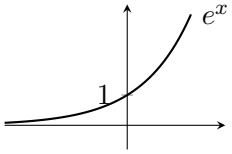
PRŮBĚH FUNKCE

$$f(x) = \frac{e^x}{x+1}$$



1) $D(f) = \mathbb{R} - \{-1\}$ znaménko $f(x)$: $\overset{-}{\qquad}\overset{+}{\qquad}$ (0 v -1)

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2)

$$f'(x) = \frac{e^x \cdot (x+1) - e^x \cdot 1}{(x+1)^2} = \frac{e^x \cdot x + e^x - e^x}{(x+1)^2} = \frac{x \cdot e^x}{(x+1)^2}$$

$f'(x) = 0$ pro $x \cdot e^x = 0$, tj. $x = 0$ znaménko $f'(x)$: $\overset{-}{\qquad}\overset{-}{\qquad}\overset{+}{\qquad}$ (0 v -1, | v 0)

rostoucí v $(0, \infty)$

klesající v $(-\infty, -1) \cup (-1, 0)$ lokální minimum $E[0,1]$: $f(0) = \frac{e^0}{0+1} = 1$

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3)

$$f''(x) = \frac{(1 \cdot e^x + x \cdot e^x)(x+1)^2 - xe^x \cdot 2(x+1)}{(x+1)^4} = \frac{(x+1)\left[(e^x + xe^x)(x+1) - 2xe^x\right]}{(x+1)^4} =$$
$$= \frac{e^x \cdot x + e^x + x^2 \cdot e^x + x \cdot e^x - 2xe^x}{(x+1)^3} = \frac{e^x(x^2+1)}{(x+1)^3}$$

$f''(x) \neq 0$ znaménko: $\overset{-}{\qquad}\overset{+}{\qquad}$ (-1)

konvexní v $(-1, \infty)$

konkávní v $(-\infty, -1)$ nemá inflexní bod

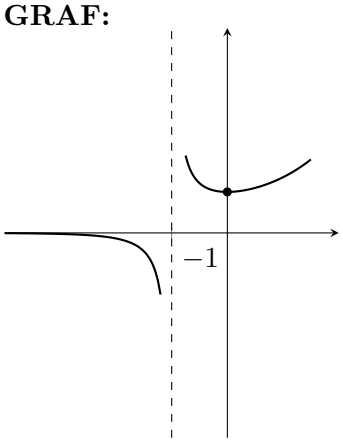
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4) asymptoty:

svislé:

$$\left.\begin{aligned} \lim_{x \to -1^+} \frac{e^x}{x+1} &= \frac{e^{-1}}{0^+} = \infty \\ \lim_{x \to -1^-} \frac{e^x}{x+1} &= \frac{e^{-1}}{0^-} = -\infty \end{aligned}\right\} \text{ svislá asymptota } x = -1$$

GRAF:



vodorovné:

$$\left.\begin{aligned} \lim_{x \to \infty} \frac{e^x}{x+1} &= \left[\frac{\infty}{\infty}\right] \overset{\text{L.P.}}{=} \lim_{x \to \infty} \frac{e^x}{1} = \infty \\ \lim_{x \to -\infty} \frac{e^x}{x+1} &= \frac{0^+}{-\infty} = 0 \end{aligned}\right\}$$

vodorovná asymptota $y = 0$ pro $x \to -\infty$

šikmá: $y = ax + b$

$$a = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{e^x}{x(x+1)} = \lim_{x \to \infty} \frac{e^x}{x^2 + x} =$$
$$= \left[\frac{\infty}{\infty}\right] \overset{\text{L.P.}}{=} \lim_{x \to \infty} \frac{e^x}{2x+1} = \left[\frac{\infty}{\infty}\right] \overset{\text{L.P.}}{=} \lim_{x \to \infty} \frac{e^x}{2} = \infty \longrightarrow \text{není}$$
$$a = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{e^x}{x(x+1)} = \frac{0^+}{\infty} = 0 \longrightarrow \text{není}$$

nemá šikmou asymptotu