Parciální zlomky – řešení

① $\boxed{\dfrac{x^2+x}{x^2+4x-12}} = 1 + \dfrac{-3x+12}{x^2+4x-12}$

- dělení:

$$\begin{array}{r} (x^2 \ \ +x) \\ -(x^2+4x-12) \\ \hline 0-3x+12 \end{array} \quad : (x^2+4x-12) = 1$$

- rozklad jmenovatele:

$$x^2+4x-12 = (x+6)(x-2) \qquad \ldots \text{ diskriminant } \ldots \text{ nebo odhadem}$$

- obecný tvar rozkladu + výpočet konstant

$$\begin{aligned} \frac{-3x+12}{(x+6)(x-2)} &= \frac{A}{x+6} + \frac{B}{x-2} \\ -3x+12 &= A(x-2) + B(x+6) \\ x=2 \ \Rightarrow \ -3\cdot 2+12 &= A\cdot 0 + B(2+6) \\ 6 = 8B \ \Rightarrow \ B &= \frac{6}{8} = \frac{3}{4} \\ x=-6 \ \Rightarrow \ -3\cdot(-6)+12 &= A(-6-2) + B\cdot 0 \\ 30 = -8A \ \Rightarrow \ A &= -\frac{30}{8} = -\frac{15}{4} \end{aligned}$$

- výsledek

$$\frac{x^2+x}{x^2+4x-12} = 1 + \frac{-\frac{15}{4}}{x+6} + \frac{\frac{3}{4}}{x-2} = \boxed{1 - \frac{15}{4(x+6)} + \frac{3}{4(x-2)}}$$

② $\boxed{\dfrac{18-5x-7x^2}{5x^3-26x^2+44x-24}}$

⊛ výsledek: $\dfrac{-2}{x-2}-\dfrac{5}{(x-2)^2}+\dfrac{3}{5x-6}$

- dělit nemusíme
- rozklad jmenovatele

$$⑤x^3-26x^2+44x-24=⑤(x-2)(x-2)\left(x-\tfrac{6}{5}\right)=\underline{(x-2)^2\cdot(⑤x-6)}$$

	5	−26	44	−24
2	5	−16	12	0

$$5x^2\ \ -16x\ \ +12\ \ \Rightarrow\ D=16^2-4\cdot5\cdot12=16$$

$$x_{1,2}=\frac{16\pm4}{2\cdot5}=\begin{cases}\frac{20}{10}=2\\ \frac{12}{10}=\frac{6}{5}\end{cases}$$

- obecný tvar rozkladu

$$\frac{18-5x-7x^2}{5x^3-26x^2+44x-24}=\frac{18-5x-7x^2}{(x-2)^2\cdot(5x-6)}=\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{5x-6}$$

- výpočet konstant

$$\frac{18-5x-7x^2}{(x-2)^2(5x-6)}=\frac{A\overbrace{(x-2)(5x-6)}^{5x^2-10x-6x+12}+B(5x-6)+C\overbrace{(x-2)^2}^{x^2-4x+4}}{(x-2)^2\cdot(5x-6)}$$

$$18-5x-7x^2=5Ax^2-16Ax+12A+5Bx-6B+Cx^2-4Cx+4C$$

$$\left.\begin{array}{rrcl}x^2: & -7 & = & 5A+C\\ x^1: & -5 & = & -16A+5B-4C\\ x^0: & 18 & = & 12A-6B+4C\end{array}\right\}\ \begin{matrix}1.\\+\\2.\end{matrix}\left\{\ 13=-4A-B\ \Rightarrow\ B=-4A-13\right.$$

$$C=-7-5A$$

do 2. rovnice:

$$-5=-16A+5(-4A-13)-4(-7-5A)\quad ⊛$$
$$-5=-16A-20A-65+28+20A$$
$$32=-16A\ \Rightarrow\ \boxed{A=-2}$$

$$\boxed{B}=-4\cdot(-2)-13=\boxed{-5},\qquad \boxed{C}=-7-5\cdot(-2)=\boxed{3}$$

③ $\boxed{\dfrac{2x+2}{(x-1)(x^2+1)^2}}$

- dělení i rozklad jmenovatele už máme
- obecný tvar rozkladu:

$$\frac{2x+2}{(x-1)(x^2+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

$$\frac{2x+2}{(x-1)(x^2+1)^2} = \frac{A\left(x^2+1\right)^2+(Bx+C)(x-1)\left(x^2+1\right)+(Dx+E)(x-1)}{(x-1)(x^2+1)^2}$$

$$2x+2 = A\left(x^4+2x^2+1\right)+(Bx+C)\left(x^3-x^2+x-1\right)+Dx^2-Dx+Ex-E$$
$$2x+2 = Ax^4+2Ax^2+A+Bx^4-Bx^3+Bx^2-Bx+Cx^3-Cx^2+Cx-C+$$
$$+Dx^2-Dx+Ex-E$$

$$\begin{array}{lrcl} x^4: & 0 & = & A+B \\ x^3: & 0 & = & -B+C \\ x^2: & 0 & = & 2A+B-C+D \\ x^1: & 2 & = & -B+C-D+E \\ x^0: & 2 & = & A-C-E \end{array} \quad \begin{array}{l} \Rightarrow\ A=-B\ \Rightarrow\ B=-1 \\ \Rightarrow\ C=B=-A\ (*)\ \Rightarrow\ C=-1 \\ \Big\} + \Big\{\ 2=2A+E\ \Rightarrow\ E=2-2A\ (**) \end{array}$$

$(*)$ a $(**)$ do poslední rovnice:

$$2 = A-(-A)-(2-2A)$$
$$2 = A+A-2+2A$$
$$4 = 4A\ \Rightarrow\ \underline{A=1}\quad (\Rightarrow B=-1,\ C=-1)$$
$$E = 2-2A = 2-2 = \underline{0}$$

dosazení do 4. rovnice (u x^1):

$$2 = -(-1)+(-1)-D+0$$
$$2 = 1-1-D\ \Rightarrow\ \underline{D=-2}$$

- výsledek

$$\boxed{\frac{2x+2}{(x-1)(x^2+1)^2} = \frac{1}{x-1} + \frac{-x-1}{x^2+1} + \frac{-2x}{(x^2+1)^2}}$$

④ $\boxed{\dfrac{x^4+6x^2+x-2}{x^4+2x^3}}\ (*) = 1 + \dfrac{-2x^3+6x^2+x-2}{x^4+2x^3}$

- dělení:

$$\begin{array}{l} (x^4 \quad\quad +6x^2+x-2) \quad : (x^4+2x^3) = 1 \\ -\underline{(x^4+2x^3)} \\ \quad\quad -2x^3+6x^2+x-2 \end{array}$$

- rozklad jmenovatele:

$$x^4+2x^3 = x^3(x+2)$$

- obecný tvar rozkladu

$$\frac{-2x^3+6x^2+x-2}{x^3(x+2)} = \frac{A}{x^3}+\frac{B}{x^2}+\frac{C}{x}+\frac{D}{x+2}$$

$$\frac{-2x^3+6x^2+x-2}{x^3(x+2)} = \frac{A(x+2)+Bx(x+2)+Cx^2(x+2)+Dx^3}{x^3(x+2)}$$

$$-2x^3+6x^2+x-2 = Ax+2A+Bx^2+2Bx+Cx^3+2Cx^2+Dx^3$$

$$\begin{array}{lrcll} x^3: & -2 & = & C+D & \Rightarrow\ -2=\frac{5}{2}+D\ \Rightarrow\ \underline{D=-2-\frac{5}{2}=-\frac{9}{2}} \\ x^2: & 6 & = & B+2C & \Rightarrow\ 2C=6-B\ \Rightarrow\ 2C=6-1\ \Rightarrow\ \underline{C=\frac{5}{2}} \\ x^1: & 1 & = & A+2B & \Rightarrow\ \boxed{2B}=1-A=1-(-1)=\boxed{2} \\ x^0: & -2 & = & 2A & \Rightarrow\ \underline{A=-1} \end{array}$$

$$\Downarrow$$

$$\underline{B=1}$$

- výsledek

$$\boxed{(*)\quad f(x) = 1-\frac{1}{x^3}+\frac{1}{x^2}+\frac{5}{2x}-\frac{9}{2(x+2)}}$$