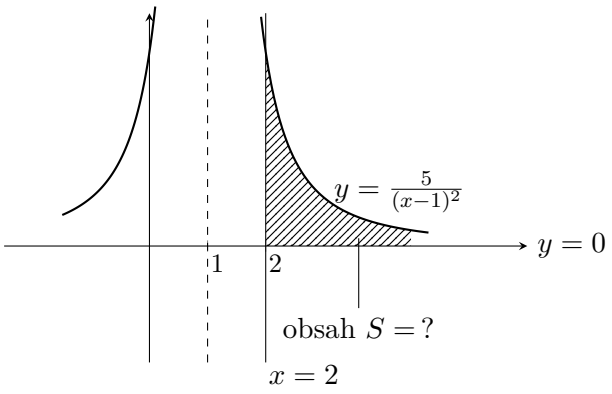
Vypočítej obsah (nakresli náčrtek):

a) $0 \le y \le \dfrac{5}{(x-1)^2}$, $\quad 2 \le x \le \infty$

$y \ge 0$ $\qquad y \le \dfrac{5}{(x-1)^2}$ $\qquad x \ge 2$ $\qquad x \le \infty$



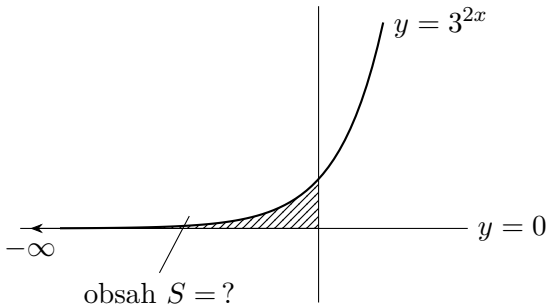
Horní − Spodní

$$S = \int_2^\infty \frac{5}{(x-1)^2} - 0 \,\mathrm{d}x = \lim_{t\to\infty} \int_2^t 5\,(x-1)^{-2}\,\mathrm{d}x = \lim_{t\to\infty} \left[5\,\frac{(x-1)^{-1}}{-1}\right]_2^t =$$

$$= \lim_{t\to\infty} \left[\frac{-5}{x-1}\right]_2^t = \lim_{t\to\infty} \left(\frac{-5}{t-1} - \left(-\frac{5}{2-1}\right)\right) = \lim_{t\to\infty} \left(\underbrace{\frac{-5}{\infty}}_{=0} + \frac{5}{1}\right) = \boxed{5}$$

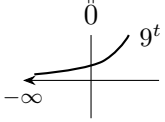
b) $0 \le y \le 3^{2x}$, $\quad -\infty \le x \le 0$

$y \ge 0$ $\qquad y \le 3^{2x} = (3^2)^x = 9^x$ $\qquad x \ge -\infty$ $\qquad x \le 0$



$$S = \int_{-\infty}^0 3^{2x} - 0\,\mathrm{d}x = \lim_{t\to-\infty} \int_t^0 \underbrace{3^{2x}}_{=9^x} \mathrm{d}x = \lim_{t\to-\infty} \left[\frac{9^x}{\ln 9}\right]_t^0 =$$

$$= \lim_{t\to-\infty} \frac{9^0}{\ln 9} - \frac{9^t}{\ln 9} = \frac{1}{\ln 9} - \frac{1}{\ln 9} \underbrace{\lim_{t\to-\infty} 9^t}_{=0} = \frac{1}{\ln 9} - \frac{1}{\ln 9}\cdot 0 = \frac{1}{\underline{\ln 9}} = \boxed{\frac{1}{2\ln 3}}$$



1. nakresli obrázek
2. přepiš pomocí nerovnic
3. urči obsah ohraničený funkcemi

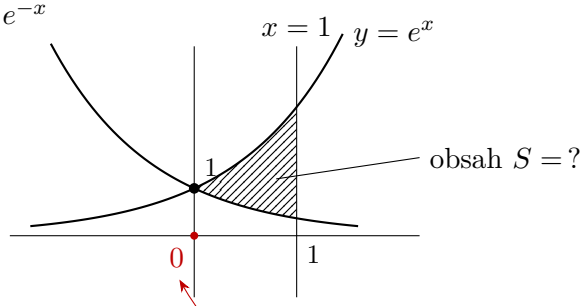
c) $f\colon\ y = e^x$

$g\colon\ y = e^{-x} = \dfrac{1}{e^x} = \left(\dfrac{1}{e}\right)^x$ ($\dfrac{1}{e} < 1$)

$x = 1$

$y = \left(\frac{1}{e}\right)^x = e^{-x}$

1)



Průsečík: $f = g$

$$\begin{aligned} e^x &= e^{-x} \quad /\ln_e \\ \ln_e e^x &= \ln_e e^{-x} \\ x \cdot \underbrace{\ln_e e}_{=1} &= -x \cdot \underbrace{\ln_e e}_{=1} \\ x &= -x \\ 2x &= 0 \\ \underline{\underline{x}} &= \underline{\underline{0}} \end{aligned}$$

2) Přepis pomocí nerovnic: $e^{-x} \le y \le e^{+x}$

$0 \le x \le 1$

3) $S = \int_0^1 e^x - e^{-x}\,\mathrm{d}x = \left[e^x - \dfrac{e^{-x}}{-1}\right]_0^1 = e^1 - \dfrac{e^{-1}}{-1} - \left(e^0 - \dfrac{e^0}{-1}\right) =$

$= e + \dfrac{1}{e} - (1+1) = \boxed{e + \dfrac{1}{e} - 2}$

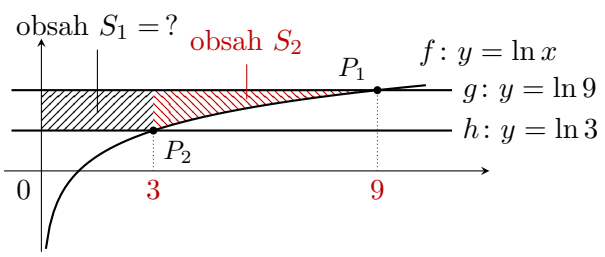
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d) $f\colon\ y = \ln x$

$g\colon\ y = \ln 9$ ⇠ konstantní funkce

$h\colon\ y = \ln 3$

$x = 0$



Průsečík P_1: $f = g$: $\ln x = \ln 9$, $\underline{\underline{x = 9}}$

P_2: $h = f$: $\ln x = \ln 3$, $\underline{\underline{x = 3}}$

! Mění se spodní funkce – PROTO je nutné POČÍTAT S_1, S_2 ZVLÁŠŤ **!**

Celkový obsah $S = S_1 + S_2 =$

$= 3\ln 3 + 6 - 3\ln 3 = \boxed{6}$

$S_1 = \int_0^3 \ln 9 - \ln 3\,\mathrm{d}x = \Big[\ln 9\, x - \ln 3\, x\Big]_0^3 = (3\ln 9 - 3\ln 3) - (0\ln 9 - 0\ln 3) =$

$= 3(\ln 9 - \ln 3) = 3\ln\dfrac{9}{3} = \underline{3\ln 3}$

$S_2 = \int_3^9 \ln 9 - \underbrace{\ln x}^{*}\,\mathrm{d}x = \Big[\ln 9\, x - (x\ln x - x)\Big]_3^9 = (9\ln 9 - 9\ln 9 + 9) - (3\ln 9 - 3\ln 3 + 3) =$

$= 9 - 3\ln 9 + 3\ln 3 - 3 = 6 - 3\ln 3^2 + 3\ln 3 = 6 - 6\ln 3 + 3\ln 3 = \underline{\underline{6 - 3\ln 3}}$

Pomocný výpočet:

$^{*}\int \ln x\,\mathrm{d}x = \int 1 \cdot \ln x\,\mathrm{d}x = \left|\begin{array}{ll} u = \ln x & u' = \frac{1}{x} \\ v' = 1 & v = x \end{array}\right| = x\ln x - \int x \cdot \frac{1}{x}\,\mathrm{d}x = \boxed{x\ln x - x}$