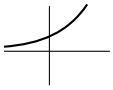
Příklady na procvičení

SUBSTITUCE:

a) $$\int_0^\infty x\cdot e^{x^2}\,\mathrm{d}x = \lim_{t\to\infty}\int_0^t x\cdot e^{x^2}\,\mathrm{d}x = \lim_{t\to\infty}\underbrace{\left[\tfrac{1}{2}e^{x^2}\right]_0^t}_{*} =$$

$$= \lim_{t\to\infty}\left(\frac{1}{2}e^{t^2}-\frac{1}{2}e^0\right) = \frac{1}{2}\lim_{t\to\infty}e^{t^2}-\frac{1}{2} = \frac{1}{2}\infty-\frac{1}{2} = \boxed{\infty} \longleftarrow \text{Integrál DIVERGUJE}$$



Pomocný výpočet

$$\int x\cdot e^{x^2}\,\mathrm{d}x = \left|\begin{array}{l} t = x^2 \\ \mathrm{d}t = 2x\,\mathrm{d}x \\ \mathrm{d}x = \frac{\mathrm{d}t}{2x}\end{array}\right| = \int \cancel{x}\cdot e^t\,\frac{\mathrm{d}t}{2\cancel{x}} = \frac{1}{2}\int e^t\,\mathrm{d}t = \frac{1}{2}e^t = \underbrace{\frac{1}{2}e^{x^2}}_{*}$$

b) $$\int_{-\infty}^{\infty}\frac{1}{x^2+2x+\mathbf{4}}\,\mathrm{d}x = \int_{-\infty}^{0}\frac{1}{(x+1)^2+3}\,\mathrm{d}x + \int_0^\infty\frac{1}{(x+1)^2+3}\,\mathrm{d}x =$$

x^2+2x+4 $\Vert$ ÚPRAVA na čtverec $\Vert$ $(x+1)^2+3$

$$= \lim_{t\to-\infty}\int_t^0\frac{1}{(x+1)^2+3}\,\mathrm{d}x + \lim_{u\to\infty}\int_0^u\frac{1}{(x+1)^2+3}\,\mathrm{d}x =$$

Pomocný výpočet

$$\int\frac{1}{(x+1)^2+3}\,\mathrm{d}x = \left|\begin{array}{l} t = x+1 \\ \mathrm{d}t = \mathrm{d}x\end{array}\right| = \int\frac{1}{t^2+3}\,\mathrm{d}t = \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{t}{\sqrt{3}} = \underbrace{\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{x+1}{\sqrt{3}}}_{**}$$

$$= \lim_{t\to-\infty}\underbrace{\left[\tfrac{1}{\sqrt{3}}\operatorname{arctg}\tfrac{x+1}{\sqrt{3}}\right]_t^0}_{**} + \lim_{u\to\infty}\underbrace{\left[\tfrac{1}{\sqrt{3}}\operatorname{arctg}\tfrac{x+1}{\sqrt{3}}\right]_0^u}_{**} =$$

$$= \lim_{t\to-\infty}\left(\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{t+1}{\sqrt{3}}\right) + \lim_{u\to\infty}\left(\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{u+1}{\sqrt{3}}-\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{1}{\sqrt{3}}\right) =$$

$$= \frac{1}{\sqrt{3}}\cdot\frac{\pi}{6}-\frac{1}{\sqrt{3}}\underbrace{\operatorname{arctg}(-\infty)}_{-\frac{\pi}{2}}+\frac{1}{\sqrt{3}}\underbrace{\operatorname{arctg}(\infty)}_{\frac{\pi}{2}}-\frac{1}{\sqrt{3}}\cdot\frac{\pi}{6} = \frac{1}{\sqrt{3}}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \boxed{\frac{\pi}{\sqrt{3}}}$$

Integrál KONVERGUJE

c) $$\int_{-\infty}^0 x^2e^{x^3}\,\mathrm{d}x = \lim_{t\to-\infty}\int_t^0 x^2e^{x^3}\,\mathrm{d}x = \lim_{t\to-\infty}\underbrace{\left[\tfrac{1}{3}e^{x^3}\right]_t^0}_{***} = \lim_{t\to-\infty}\left[\frac{1}{3}-\frac{e^{t^3}}{3}\right] = \frac{1}{3}-\frac{e^{-\infty}}{3} =$$

Pomocný výpočet

$$\int x^2e^{x^3}\,\mathrm{d}x = \left|\begin{array}{l} t = x^3 \\ \mathrm{d}t = 3x^2\,\mathrm{d}x \\ \frac{\mathrm{d}t}{3x^2} = \mathrm{d}x\end{array}\right| = \int \cancel{x^2}e^t\,\frac{\mathrm{d}t}{3\cancel{x^2}} = \frac{1}{3}e^t = \underbrace{\frac{1}{3}e^{x^3}}_{***}$$

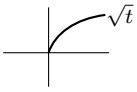
$$= \frac{1}{3}-\frac{1}{3e^\infty} =$$

$$= \frac{1}{3}-\underbrace{\frac{1}{3\infty}}_{\Vert\ 0} = \boxed{\frac{1}{3}}$$

KONVERG.

a) $$\int_0^\infty 4\sqrt{x}\,\mathrm{d}x = \lim_{t\to\infty}\int_0^t 4x^{\frac{1}{2}}\,\mathrm{d}x = \lim_{t\to\infty}\left[4\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^t =$$

$$= \lim_{t\to\infty}\left[\frac{8\sqrt{x^3}}{3}\right]_0^t = \lim_{t\to\infty}\left(\frac{8\sqrt{t^3}}{3} - 0\right) = \boxed{\infty}$$



Integrál diverguje

b) $$\int_2^\infty \frac{1}{x^2}\,\mathrm{d}x = \lim_{t\to\infty}\int_2^t x^{-2}\,\mathrm{d}x = \lim_{t\to\infty}\left[\frac{x^{-1}}{-1}\right]_2^t = \lim_{t\to\infty}\left[-\frac{1}{x}\right]_2^t =$$

$$= \lim_{t\to\infty}\left(-\frac{1}{t} - \left(-\frac{1}{2}\right)\right) = \underset{\|\ 0}{\left(-\frac{1}{\infty}\right)} + \frac{1}{2} = \boxed{\frac{1}{2}}$$

Integrál KONVERGUJE

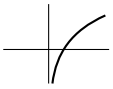
c) $$\int_1^\infty \frac{x^3-1}{x^4}\,\mathrm{d}x = \lim_{t\to\infty}\int_1^t \frac{x^3-1}{x^4}\,\mathrm{d}x = \lim_{t\to\infty}\int_1^t \frac{1}{x} - x^{-4}\,\mathrm{d}x =$$

$$= \lim_{t\to\infty}\left[\ln|x| - \frac{x^{-3}}{-3}\right]_1^t = \lim_{t\to\infty}\left[\ln|x| + \frac{1}{3x^3}\right]_1^t =$$

$$= \lim_{t\to\infty}\left(\underset{\|\ \infty}{\ln|t|} + \underset{\|\ 0}{\frac{1}{3t^3}} - \left(\underset{\|\ 0}{\ln|1|} + \frac{1}{3}\right)\right) =$$

$$= \infty + 0 - 0 - \frac{1}{3} = \boxed{\infty}$$

diverguje



PER PARTES: + limita $\frac{\infty}{\infty}$ = L'Hospitalovo pravidlo (L.P.)

a) $$\int_0^\infty 4x\cdot e^{-2x}\,\mathrm{d}x = \lim_{t\to\infty}\int_0^t 4x\cdot e^{-2x}\,\mathrm{d}x = \lim_{t\to\infty}\underbrace{\left[\frac{-2x-1}{e^{2x}}\right]_0^t}_{*} =$$

$$= \lim_{t\to\infty}\left(\frac{-2t-1}{e^{2t}} - \left(\frac{-1}{e^0}\right)\right) = \frac{-2}{e^{2t}\cdot 2} + \frac{1}{1} = \underset{\|\ 0}{\left(-\frac{1}{\infty}\right)} + 1 = \boxed{1}$$

$\frac{\infty}{\infty}$! L.P.!



L.P.: $\frac{(-2t-1)'}{(e^{2t})'} = \frac{-2}{e^{2t}\cdot 2}$

Integrál KONVERGUJE

Pomocný výpočet

$$\int 4x\cdot e^{-2x}\,\mathrm{d}x = \left|\begin{array}{ll} u = 4x & \to\ u' = 4 \\ v' = e^{-2x} & \quad\ v = \frac{e^{-2x}}{-2} \end{array}\right| = \cancel{4}^{\,2}x\left(\frac{1}{\cancel{-2}e^{2x}}\right) - \int \frac{\cancel{4}^{\,2}}{\cancel{-2}\,e^{2x}}\,\mathrm{d}x =$$

$$= -\frac{2x}{e^{2x}} + 2\int e^{-2x}\,\mathrm{d}x = -\frac{2x}{e^{2x}} + \cancel{2}\,\frac{e^{-2x}}{\cancel{-2}} = \underbrace{-\frac{2x}{e^{2x}} - \frac{1}{e^{2x}}}_{*}$$