Integruj:

1)

$$\int \frac{-2}{x+5}\,\mathrm{d}x = -2\int \frac{1}{x+5}\,\mathrm{d}x = \boxed{-2\ln|x+5|+c}$$

derivace spodku je vršek

2)

$$\int \frac{-2x}{x+5}\,\mathrm{d}x = -2\int \frac{x+5-5}{x+5}\,\mathrm{d}x = -2\int\left(\frac{x+5}{x+5}-\frac{5}{x+5}\right)\mathrm{d}x =$$

$$= -2\left(\int 1\,\mathrm{d}x - 5\int \frac{1}{x+5}\,\mathrm{d}x\right) = -2\left(x-5\ln|x+5|\right) =$$

derivace spodku je vršek

$$= \boxed{-2x+10\ln|x+5|+c}$$

3)

$$\int \frac{-2x+\mathbf{3}}{x+5}\,\mathrm{d}x = \int \frac{-2x+3}{x+5}\,\mathrm{d}x = \int\left(\frac{-2x}{x+5}+\frac{3}{x+5}\right)\mathrm{d}x =$$

$$= -2\int \frac{x+5-5}{x+5}\,\mathrm{d}x + 3\int \frac{1}{x+5}\,\mathrm{d}x =$$

$$= -2\int\left(\frac{x+5}{x+5}-\frac{5}{x+5}\right)\mathrm{d}x + 3\ln|x+5| = -2\left(x-5\ln|x+5|\right)+3\ln|x+5| =$$

$a\cdot\log x = \log x^a$

$$= -2x+10\ln|x+5|+3\ln|x+5| = -2x+\ln|x+5|^{10}+\ln|x+5|^3 =$$

$$= -2x+\ln\left(|x+5|^{10}\cdot|x+5|^3\right) = \boxed{-2x+\ln|x+5|^{13}+c}$$

$\log_z a+\log_z b = \log_z(a\cdot b)$
$\log_z a-\log_z b = \log_z \frac{a}{b}$

} Musí mít logaritmy stejný základ! (= malé písmeno z)

4)

$$\int \frac{-3}{(x+5)^2}\,\mathrm{d}x = -3\int \frac{1}{(x+5)^2}\,\mathrm{d}x = \left|\begin{array}{l} x+5=t \\ \mathrm{d}x=\mathrm{d}t \end{array}\right| = -3\int \frac{1}{t^2}\,\mathrm{d}t = -3\int t^{-2}\,\mathrm{d}t =$$

$$= -3\,\frac{t^{-1}}{-1}+c = +3\,\frac{1}{t}+c = \boxed{\frac{3}{x+5}+c}$$

5)

$$\int \frac{-3\mathbf{x}}{(x+5)^2}\,\mathrm{d}x = \int\left(\frac{A}{(x+5)^2}+\frac{B}{(x+5)^1}\right)\mathrm{d}x = \int\left(\frac{15}{(x+5)^2}+\frac{-3}{x+5}\right)\mathrm{d}x =$$

$/\cdot(x+5)^2$

$$-3x = A+B(x+5)$$
$$-3x = A+Bx+5B$$
$$x^1: \quad \boxed{-3=B}$$
$$x^0: \quad 0=A+5B \;\rightarrow\; 0=A+5\cdot(-3) \;\Rightarrow\; \boxed{A=15}$$

$$= 15\int (x+5)^{-2}\,\mathrm{d}x - 3\int \frac{1}{x+5}\,\mathrm{d}x = 15\cdot\frac{(x+5)^{-1}}{-1}-3\ln|x+5| = \boxed{\frac{-15}{x+5}-3\ln|x+5|+c}$$

$= \ln|x+5|$

6)

není „x“, proto jen upravím na čtverec

$$\int \frac{6}{x^2+x+\mathbf{3}}\,\mathrm{d}x = \int \frac{6}{\left(x+\frac{1}{2}\right)^2-\frac{1}{4}+3}\,\mathrm{d}x = \int \frac{6}{\left(x+\frac{1}{2}\right)^2+\frac{11}{4}}\,\mathrm{d}x = 6\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{11}{4}}\,\mathrm{d}x =$$

$D = 1^2 - 4\cdot 1\cdot 3 < 0$

$\frac{12}{4}$

$$= 6\cdot\frac{2}{\sqrt{11}}\operatorname{arctg}\frac{\left(x+\frac{1}{2}\right)}{\frac{\sqrt{11}}{2}} = \frac{12}{\sqrt{11}}\operatorname{arctg}\frac{\left(x+\frac{1}{2}\right)}{\frac{\sqrt{11}}{2}} = \boxed{\frac{12}{\sqrt{11}}\operatorname{arctg}\frac{2x+1}{\sqrt{11}}+c}$$

7)

je „x“, proto nahoru píšu derivaci spodku

$$\int \frac{6x+\mathbf{4}}{x^2-4x+8}\,\mathrm{d}x = \int \frac{3(2x-4)+12+4}{x^2-4x+8}\,\mathrm{d}x = \int \frac{3(2x-4)+16}{x^2-4x+8}\,\mathrm{d}x =$$

$D = (-4)^2 - 4\cdot 1\cdot 8 < 0$

$$= 3\int \frac{2x-4}{x^2-4x+8}\,\mathrm{d}x + 16\int \frac{1}{x^2-4x+8} =$$

$= \ln\left|x^2-4x+8\right|$

$=$ úprava na čtverec

$\|$

$$\int \frac{1}{(x-2)^2-4+8}\,\mathrm{d}x =$$

$$= \int \frac{1}{(x-2)^2+4}\,\mathrm{d}x = \frac{1}{2}\operatorname{arctg}\frac{x-2}{2}$$

$$= \boxed{3\ln\left|x^2-4x+8\right| + 16\cdot\frac{1}{2}\operatorname{arctg}\frac{x-2}{2}+c}$$

8)

$$\int \frac{5x+2}{x^5+x^3}\,\mathrm{d}x = \int \frac{5x+2}{x^3(x^2+1)}\,\mathrm{d}x\,; \qquad \underbrace{\frac{5x+2}{x^3(x^2+1)} = \frac{A}{x^3}+\frac{B}{x^2}+\frac{C}{x}+\frac{Dx+E}{x^2+1}}$$

$$\boxed{D = 0^2 - 4\cdot 1\cdot 1 < 0} \qquad \Big/ \cdot x^3(x^2+1)$$

$$5x+2 = A(x^2+1)+Bx(x^2+1)+Cx^2(x^2+1)+(Dx+E)x^3$$

$$5x+2 = Ax^2+A+Bx^3+Bx+Cx^4+Cx^2+Dx^4+Ex^3$$

$$\begin{aligned}
x^4:&\quad 0 = C+D \;\longrightarrow\; 0 = -2+D \;\Rightarrow\; \boxed{D=2}\\
x^3:&\quad 0 = B+E \;\longrightarrow\; 0 = 5+E \;\Rightarrow\; \boxed{E=-5}\\
x^2:&\quad 0 = A+C \;\to\; 0 = 2+C \;\Rightarrow\; \boxed{C=-2}\\
x^1:&\quad \boxed{5=B}\\
x^0:&\quad \boxed{2=A}
\end{aligned}$$

$$\int \left(\frac{2}{x^3}+\frac{5}{x^2}+\frac{-2}{x}+\frac{2x-5}{x^2+1}\right)\mathrm{d}x =$$

$$= 2\int x^{-3}\,\mathrm{d}x + 5\int x^{-2}\,\mathrm{d}x - 2\underbrace{\int \frac{1}{x}\,\mathrm{d}x}_{=\ln|x|} + \underbrace{\int \frac{2x}{x^2+1}\,\mathrm{d}x}_{=\ln|x^2+1|} - 5\int \frac{1}{x^2+1}\,\mathrm{d}x =$$

$$= 2\,\frac{x^{-2}}{-2} + 5\,\frac{x^{-1}}{-1} - 2\ln|x| + \ln\left|x^2+1\right| - 5\,\mathrm{arctg}\,\frac{x}{1} =$$

$$= \boxed{-\frac{1}{x^2}-\frac{5}{x}-2\ln|x|+\ln\left|x^2+1\right|-5\,\mathrm{arctg}\,x+c}$$