Integruj = Najdi primitivní funkci:

1.

$$\int 4\sqrt{x^5}\,\mathrm{d}x = 4\int \sqrt{x^5}\,\mathrm{d}x = 4\int x^{\frac{5}{2}}\,\mathrm{d}x = 4\,\frac{x^{\frac{7}{2}}}{\frac{7}{2}} = \frac{4\cdot 2\sqrt{x^7}}{7} = \boxed{\frac{8x^3\sqrt{x}}{7} + c}$$

2.

$$\int 3\,\frac{\mathrm{d}x}{\sin^2 x} = 3\int \frac{1}{\sin^2 x}\,\mathrm{d}x = 3\cdot(-\operatorname{cotg} x) + c = \boxed{-3\operatorname{cotg} x + c}$$

3.

$$\int (x+1)\cdot(x^2+2x)\,\mathrm{d}x = \int \left(x^3+2x^2+x^2+2x\right)\,\mathrm{d}x = \int \left(x^3+3x^2+2x\right)\,\mathrm{d}x =$$

$$= \frac{x^4}{4} + \cancel{3}\,\frac{x^3}{\cancel{3}} + \frac{\cancel{2}x^2}{\cancel{2}} + c = \boxed{\frac{x^4}{4} + x^3 + x^2 + c}$$

4.

$$\int \frac{x\sqrt[3]{x} - 2\sqrt[5]{x^2}}{x\sqrt{x}}\,\mathrm{d}x = \int \left(x\cdot x^{\frac{1}{3}}\cdot x^{-1}\cdot x^{-\frac{1}{2}} - 2x^{\frac{2}{5}}\cdot x^{-1}\cdot x^{-\frac{1}{2}}\right)\,\mathrm{d}x =$$

$$= \int \left(x^{1+\frac{1}{3}-1-\frac{1}{2}} - 2x^{\frac{2}{5}-1-\frac{1}{2}}\right)\,\mathrm{d}x = \int \left(x^{\frac{6+2-6-3}{6}} - 2x^{\frac{4-10-5}{10}}\right)\,\mathrm{d}x =$$

$$= \int \left(x^{-\frac{1}{6}} - 2x^{-\frac{11}{10}}\right)\,\mathrm{d}x = \frac{x^{\frac{5}{6}}}{\frac{5}{6}} - 2\,\frac{x^{-\frac{1}{10}}}{-\frac{1}{10}} + c = \boxed{\frac{6\sqrt[6]{x^5}}{5} + \frac{20}{\sqrt[10]{x}} + c}$$

$$-\tfrac{1}{6}+\tfrac{6}{6} = \tfrac{5}{6} \qquad -\tfrac{11}{10}+\tfrac{10}{10} = -\tfrac{1}{10}$$

5.

$$\int (4\cdot 5^x - 2\cos x + e)\,\mathrm{d}x = 4\int 5^x\,\mathrm{d}x - 2\int \cos x\,\mathrm{d}x + e\int \mathrm{d}x = \boxed{4\,\frac{5^x}{\ln 5} - 2\sin x + ex + c}$$

6.

$$\int x^3\cdot\ln x\,\mathrm{d}x = \left|\begin{array}{lll} u = \ln x & \to & u' = \frac{1}{x} \\ v' = x^3 & \to & v = \frac{x^4}{4} \end{array}\right| = \frac{x^4}{4}\ln x - \int \frac{1}{\cancel{x}}\cdot\frac{x^{\cancel{4}3}}{4}\,\mathrm{d}x =$$

$$= \frac{x^4\ln x}{4} - \frac{1}{4}\int x^3\,\mathrm{d}x = \frac{x^4\ln x}{4} - \frac{1}{4}\cdot\frac{x^4}{4} + c =$$

$$= \frac{x^4\ln x}{4} - \frac{x^4}{16} + c = \boxed{\frac{x^4}{4}\left(\ln x - \frac{1}{4}\right) + c}$$

7.

$$\int x^2\cdot\sin x\,\mathrm{d}x = \left|\begin{array}{lll} u = x^2 & \to & u' = 2x \\ v' = \sin x & \to & v = -\cos x \end{array}\right| = x^2(-\cos x) - \int 2x\cdot(-\cos x)\,\mathrm{d}x =$$

$$= -x^2\cos x + 2\int x\cdot\cos x\,\mathrm{d}x = \left|\begin{array}{lll} u = x & \to & u' = 1 \\ v' = \cos x & \to & v = \sin x \end{array}\right| =$$

$$= -x^2\cos x + 2\left[x\sin x - \int \sin x\,\mathrm{d}x\right] =$$

$$= -x^2\cos x + 2x\sin x - 2(-\cos x) + c = \boxed{-x^2\cos x + 2x\sin x + 2\cos x + c}$$

8.

$$\int (6x-3)^7\,\mathrm{d}x = \left|\begin{array}{l} 6x-3=t \\ 6\,\mathrm{d}x=\mathrm{d}t \\ \mathrm{d}x=\frac{\mathrm{d}t}{6}\end{array}\right| = \int t^7\,\frac{\mathrm{d}t}{6} = \frac{1}{6}\int t^7\,\mathrm{d}t = \frac{1}{6}\cdot\frac{t^8}{8}+c = \frac{t^8}{48}+c = \boxed{\frac{(6x-3)^8}{48}+c}$$

9.

$$\int x\sqrt{2-x^2}\,\mathrm{d}x = \left|\begin{array}{l} 2-x^2=t \\ -2x\,\mathrm{d}x=\mathrm{d}t \\ \mathrm{d}x=\frac{\mathrm{d}t}{-2x}\end{array}\right| = \int \cancel{x}\sqrt{t}\,\frac{\mathrm{d}t}{-2\cancel{x}} = -\frac{1}{2}\int t^{\frac{1}{2}}\,\mathrm{d}t =$$

$$= -\frac{1}{2}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+c = -\frac{\cancel{2}\sqrt{(2-x^2)^3}}{\cancel{2}\cdot 3}+c = \boxed{-\frac{(2-x^2)\sqrt{2-x^2}}{3}+c}$$

10.

$$\int \frac{\ln x}{x}\,\mathrm{d}x = \left|\begin{array}{l} \ln x=t \\ \frac{1}{x}\,\mathrm{d}x=\mathrm{d}t \\ \mathrm{d}x=x\,\mathrm{d}t\end{array}\right| = \int \frac{t}{\cancel{x}}\cancel{x}\mathrm{d}t = \frac{t^2}{2}+c = \boxed{\frac{(\ln x)^2}{2}+c}$$