DERIVACE FUNKCE

① $y = e^{\sqrt{x^2+x+1}}$

$$y' = e^{\sqrt{x^2+x+1}} \cdot \left(\sqrt{x^2+x+1}\right)' = e^{\sqrt{x^2+x+1}} \cdot \left((x^2+x+1)^{\frac{1}{2}}\right)' =$$

$$= e^{\sqrt{x^2+x+1}} \cdot \frac{1}{2}(x^2+x+1)^{\frac{1}{2}-1} \cdot (x^2+x+1)' =$$

$$= e^{\sqrt{x^2+x+1}} \cdot \frac{1}{2}(x^2+x+1)^{-\frac{1}{2}} \cdot (2x+1) =$$

$$= \frac{(2x+1)\cdot e^{\sqrt{x^2+x+1}}}{2\sqrt{x^2+x+1}}$$

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② $y = \sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}} \quad \left(= \left(\operatorname{arctg} \ln \frac{x+1}{x+2}\right)^{\frac{1}{2}} \right)$

$$y' = \frac{1}{2}\left(\operatorname{arctg} \ln \frac{x+1}{x+2}\right)^{\frac{1}{2}-1} \cdot \left(\operatorname{arctg} \ln \frac{x+1}{x+2}\right)' =$$

$$= \frac{1}{2}\left(\operatorname{arctg} \ln \frac{x+1}{x+2}\right)^{-\frac{1}{2}} \cdot \frac{1}{1+\left(\ln \frac{x+1}{x+2}\right)^2} \cdot \left(\ln \frac{x+1}{x+2}\right)' =$$

$$= \frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}}} \cdot \frac{1}{1+\ln^2 \frac{x+1}{x+2}} \cdot \frac{1}{\frac{x+1}{x+2}} \cdot \left(\frac{x+1}{x+2}\right)' =$$

$$= \frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}}} \cdot \frac{1}{1+\ln^2 \frac{x+1}{x+2}} \cdot \frac{x+2}{x+1} \cdot \frac{(x+1)'\cdot(x+2)-(x+1)\cdot(x+2)'}{(x+2)^2}$$

$$= \frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}}} \cdot \frac{1}{1+\ln^2 \frac{x+1}{x+2}} \cdot \frac{\cancel{x+2}}{x+1} \cdot \frac{1\cdot(x+2)-(x+1)\cdot 1}{(x+2)^{\cancel{2}}} =$$

$$= \frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}}} \cdot \frac{1}{1+\ln^2 \frac{x+1}{x+2}} \cdot \frac{1}{x+1} \cdot \frac{\overbrace{x+2-x-1}^{1}}{x+2} =$$

$$= \frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}} \cdot \left(1+\ln^2 \frac{x+1}{x+2}\right) \cdot (x+1)(x+2)}$$

(3) $y = \ln \dfrac{1}{x+\sqrt{x^2-1}}$

$$\overbrace{\left(x+\sqrt{x^2-1}\right)^{-1}}$$

$$y' = \cancel{\frac{x}{x+\sqrt{x^2-1}}} = \frac{1}{\dfrac{1}{x+\sqrt{x^2-1}}} \cdot \left(\frac{1}{x+\sqrt{x^2-1}}\right)' =$$

$$= \left(x+\sqrt{x^2-1}\right) \cdot (-1)\left(x+\sqrt{x^2-1}\right)^{-1-1} \cdot \Big(x+\overbrace{\sqrt{x^2-1}}^{(x^2-1)^{\frac{1}{2}}}\Big)' =$$

$$= \left(x+\sqrt{x^2-1}\right) \cdot (-1)\left(x+\sqrt{x^2-1}\right)^{-2} \cdot \left(1 + \tfrac{1}{2}(x^2-1)^{\frac{1}{2}-1} \cdot (x^2-1)'\right) =$$

$$= \frac{\cancel{x+\sqrt{x^2-1}}}{-1 \cdot \left(x+\sqrt{x^2-1}\right)^{\cancel{2}}} \cdot \left(1 + \tfrac{1}{\cancel{2}}(x^2-1)^{-\frac{1}{2}} \cdot \cancel{2}x\right) =$$

$$= \frac{-1}{x+\sqrt{x^2-1}} \cdot \left(1 + \frac{x}{\sqrt{x^2-1}}\right) = \frac{-1}{\cancel{x+\sqrt{x^2-1}}} \cdot \frac{\cancel{\sqrt{x^2-1}+x}}{\sqrt{x^2-1}} =$$

$$= \underline{\underline{\frac{-1}{\sqrt{x^2-1}}}}$$

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(4) $y = \ln \operatorname{arctg} \dfrac{\sin x}{1+\cos x}$

$$y' = \frac{1}{\operatorname{arctg} \dfrac{\sin x}{1+\cos x}} \cdot \left(\operatorname{arctg} \frac{\sin x}{1+\cos x}\right)' =$$

$$= \frac{1}{\operatorname{arctg} \dfrac{\sin x}{1+\cos x}} \cdot \frac{1}{1+\left(\dfrac{\sin x}{1+\cos x}\right)^2} \cdot \left(\frac{\sin x}{1+\cos x}\right)' =$$

$$= \frac{1}{\operatorname{arctg} \dfrac{\sin x}{1+\cos x}} \cdot \frac{1}{1+\dfrac{\sin^2 x}{(1+\cos x)^2}} \cdot \frac{(\sin x)'(1+\cos x) - \sin x\,(1+\cos x)'}{(1+\cos x)^2} =$$

$$= \frac{1}{\operatorname{arctg} \dfrac{\sin x}{1+\cos x}} \cdot \frac{1}{\dfrac{(1+\cos x)^2+\sin^2 x}{\cancel{(1+\cos x)^2}}} \cdot \frac{\cos x\,(1+\cos x) - \sin x\,(-\sin x)}{\cancel{(1+\cos x)^2}} =$$

$$= \frac{1}{\operatorname{arctg} \dfrac{\sin x}{1+\cos x}} \cdot \frac{\cos x + \overbrace{\cos^2 x + \sin^2 x}^{1}}{1+2\cos x + \underbrace{\cos^2 x + \sin^2 x}_{1}} = \longrightarrow$$

pokračování (4)

$$= \frac{1}{\operatorname{arctg}\frac{\sin x}{1+\cos x}} \cdot \frac{\cos x + 1}{\underbrace{2+2\cos x}_{2(1+\cos x)}} = \frac{1}{2\operatorname{arctg}\frac{\sin x}{1+\cos x}}$$

(5) $y = \frac{1}{\log(3x^2+x+1)} = \left(\log(3x^2+x+1)\right)^{-1}$

$$y' = (-1)\cdot\left(\log(3x^2+x+1)\right)^{-2} \cdot \left(\log(3x^2+x+1)\right)' =$$

$$= \frac{-1}{\log^2(3x^2+x+1)} \cdot \frac{1}{(3x^2+x+1)\cdot \ln 10} \cdot (3x^2+x+1)' =$$

$$= \frac{-(6x+1)}{(3x^2+x+1)\cdot \ln 10 \cdot \log^2(3x^2+x+1)}$$

(6) $y = 5^{x^2-2x+1}$

$$y' = 5^{x^2-2x+1} \cdot \ln 5 \cdot (x^2-2x+1)' = (2x-2)\ln 5 \cdot 5^{x^2-2x+1}$$

(7) $y = \frac{\cos x}{2\sin^2 x}$

$$y' = \frac{(\cos x)' \, 2\sin^2 x - \cos x \, (2\underbrace{\sin^2 x}_{(\sin x)^2})'}{(2\sin^2 x)^2} =$$

$$= \frac{-\sin x \cdot 2\sin^2 x - \cos x \cdot 2 \cdot 2(\sin x)^{2-1} \cdot (\sin x)'}{4\sin^4 x} =$$

$$= \frac{-2\sin^3 x - 4\cos x \cdot \sin x \cdot (\cos x)}{4\sin^4 x} =$$

$1 = \cos^2 x + \cos^2 x$

$$= \frac{-2\sin^3 x - 4\cos^2 x \sin x}{4\sin^4 x} = \frac{-2\sin x\,(\sin^2 x + 2\cos^2 x)}{4\sin^4 x} = -\frac{1+\cos^2 x}{2\sin^3 x}$$

⑧ $y=\frac{2x^3}{\sqrt{x}}+\frac{x}{2\sqrt[3]{x}}=2\frac{x^3}{x^{\frac{1}{2}}}+\frac{1}{2}\frac{x}{x^{\frac{1}{3}}}=2x^{\frac{5}{2}}+\frac{1}{2}x^{\frac{2}{3}}$

jen úpravy mocnin, pak až derivovat

$y'=2\cdot\frac{5}{2}x^{\frac{5}{2}-1}+\frac{1}{2}\cdot\frac{2}{3}x^{\frac{2}{3}-1}=$

$=5x^{\frac{3}{2}}+\frac{1}{3}x^{-\frac{1}{3}}=\underline{\underline{5\sqrt{x^3}+\frac{1}{3\sqrt[3]{x}}}}$

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⑨ $y=\frac{x}{\sqrt{1-x^2}}\cdot\arcsin x$

$y'=\left(\frac{x}{\sqrt{1-x^2}}\right)'\cdot\arcsin x+\frac{x}{\sqrt{1-x^2}}\cdot(\arcsin x)'=$

$=\frac{(x)'\sqrt{1-x^2}-x\cdot(\overset{(1-x^2)^{\frac{1}{2}}}{\sqrt{1-x^2}})'}{(\sqrt{1-x^2})^2}\cdot\arcsin x+\frac{x}{\sqrt{1-x^2}}\cdot\frac{1}{\sqrt{1-x^2}}=$

$=\frac{1\cdot\sqrt{1-x^2}-x\cdot\frac{1}{2}(1-x^2)^{\overset{-\frac{1}{2}}{\frac{1}{2}-1}}\cdot(1-x^2)'}{1-x^2}\cdot\arcsin x+\frac{x}{1-x^2}=$

$=\frac{\sqrt{1-x^2}-\frac{x}{2\sqrt{1-x^2}}\cdot(-2x)}{1-x^2}\cdot\arcsin x+\frac{x}{1-x^2}=$

$=\frac{\sqrt{1-x^2}+\frac{2x^2}{2\sqrt{1-x^2}}}{1-x^2}\cdot\arcsin x+\frac{x}{1-x^2}=$

$=\frac{\frac{1-x^2+x^2}{\sqrt{1-x^2}}}{1-x^2}\cdot\arcsin x+\frac{x}{1-x^2}=\underline{\underline{\frac{1}{\sqrt{1-x^2}\cdot(1-x^2)}\cdot\arcsin x+\frac{x}{1-x^2}}}$

lze ještě upravit takto: $\frac{\arcsin x+x\cdot\sqrt{1-x^2}}{\sqrt{1-x^2}\cdot(1-x^2)}=\underline{\underline{\frac{\arcsin x+x\cdot\sqrt{1-x^2}}{\sqrt{(1-x^2)^3}}}}$

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⑩ $y=\operatorname{tg}\sqrt{x}+\operatorname{tg}\frac{\pi}{4}$ konstanta

$y'=\frac{1}{\cos^2\sqrt{x}}\cdot(\underbrace{\sqrt{x}}_{x^{\frac{1}{2}}})'+0=\frac{1}{\cos^2\sqrt{x}}\cdot\frac{1}{2}\cdot x^{-\frac{1}{2}}=\underline{\underline{\frac{1}{2\sqrt{x}\cos^2\sqrt{x}}}}$