DERIVACE FUNKCE

① $y = e^{\sqrt{x^2+x+1}}$

$$y' = e^{\sqrt{x^2+x+1}} \cdot \left(\sqrt{x^2+x+1}\right)' = e^{\sqrt{x^2+x+1}} \cdot \left(\left(x^2+x+1\right)^{\frac{1}{2}}\right)' =$$
$$= e^{\sqrt{x^2+x+1}} \cdot \frac{1}{2}\left(x^2+x+1\right)^{\frac{1}{2}-1} \cdot \left(x^2+x+1\right)' =$$
$$= e^{\sqrt{x^2+x+1}} \cdot \frac{1}{2}\left(x^2+x+1\right)^{-\frac{1}{2}} \cdot (2x+1) = \boxed{\frac{(2x+1)\cdot e^{\sqrt{x^2+x+1}}}{2\sqrt{x^2+x+1}}}$$

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② $y = \sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}} \quad \left(= \left(\operatorname{arctg} \ln \frac{x+1}{x+2}\right)^{\frac{1}{2}}\right)$

$$y' = \frac{1}{2}\left(\operatorname{arctg} \ln \frac{x+1}{x+2}\right)^{\frac{1}{2}-1} \cdot \left(\operatorname{arctg} \ln \frac{x+1}{x+2}\right)' =$$
$$= \frac{1}{2}\left(\operatorname{arctg} \ln \frac{x+1}{x+2}\right)^{-\frac{1}{2}} \cdot \frac{1}{1+\left(\ln \frac{x+1}{x+2}\right)^2} \cdot \left(\ln \frac{x+1}{x+2}\right)' =$$
$$= \frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}}} \cdot \frac{1}{1+\ln^2 \frac{x+1}{x+2}} \cdot \frac{1}{\frac{x+1}{x+2}} \cdot \left(\frac{x+1}{x+2}\right)' =$$
$$= \frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}}} \cdot \frac{1}{1+\ln^2 \frac{x+1}{x+2}} \cdot \frac{x+2}{x+1} \cdot \frac{(x+1)'\cdot(x+2) - (x+1)\cdot(x+2)'}{(x+2)^2} =$$
$$= \frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}}} \cdot \frac{1}{1+\ln^2 \frac{x+1}{x+2}} \cdot \cancel{\frac{x+2}{x+1}} \cdot \frac{1\cdot(x+2) - (x+1)\cdot 1}{(x+2)^{\cancel{2}}} =$$
$$= \frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}}} \cdot \frac{1}{1+\ln^2 \frac{x+1}{x+2}} \cdot \frac{1}{x+1} \cdot \frac{\overbrace{x+2-x-1}^{1}}{x+2} =$$
$$= \boxed{\frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}} \cdot \left(1+\ln^2 \frac{x+1}{x+2}\right)\cdot(x+1)(x+2)}}$$

③ $y = \ln \frac{1}{x+\sqrt{x^2-1}}$

$$y' = \frac{1}{\frac{1}{x+\sqrt{x^2-1}}} \cdot \Big(\underbrace{\frac{1}{x+\sqrt{x^2-1}}}_{\left(x+\sqrt{x^2-1}\right)^{-1}}\Big)' =$$
$$= \left(x+\sqrt{x^2-1}\right)\cdot(-1)\left(x+\sqrt{x^2-1}\right)^{-1-1} \cdot \Big(x+\underbrace{\sqrt{x^2-1}}_{(x^2-1)^{\frac{1}{2}}}\Big)' =$$
$$= \left(x+\sqrt{x^2-1}\right)\cdot(-1)\left(x+\sqrt{x^2-1}\right)^{-2} \cdot \left(1+\frac{1}{2}\left(x^2-1\right)^{\frac{1}{2}-1}\cdot\left(x^2-1\right)'\right) =$$
$$= \frac{\cancel{x+\sqrt{x^2-1}}}{-1\cdot\left(x+\sqrt{x^2-1}\right)^{\cancel{2}}} \cdot \left(1+\frac{1}{2}\left(x^2-1\right)^{-\frac{1}{2}}\cdot 2x\right) =$$
$$= \frac{-1}{x+\sqrt{x^2-1}} \cdot \left(1+\frac{x}{\sqrt{x^2-1}}\right) = \frac{-1}{\cancel{x+\sqrt{x^2-1}}} \cdot \frac{\cancel{\sqrt{x^2-1}+x}}{\sqrt{x^2-1}} = \boxed{\frac{-1}{\sqrt{x^2-1}}}$$

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④ $y = \ln \operatorname{arctg} \dfrac{\sin x}{1+\cos x}$

$$y' = \frac{1}{\operatorname{arctg} \frac{\sin x}{1+\cos x}} \cdot \left(\operatorname{arctg} \frac{\sin x}{1+\cos x}\right)' = \frac{1}{\operatorname{arctg} \frac{\sin x}{1+\cos x}} \cdot \frac{1}{1+\left(\frac{\sin x}{1+\cos x}\right)^2} \cdot \left(\frac{\sin x}{1+\cos x}\right)' =$$

$$= \frac{1}{\operatorname{arctg} \frac{\sin x}{1+\cos x}} \cdot \frac{1}{1+\frac{\sin^2 x}{(1+\cos x)^2}} \cdot \frac{(\sin x)'(1+\cos x) - \sin x(1+\cos x)'}{(1+\cos x)^2} =$$

$$= \frac{1}{\operatorname{arctg} \frac{\sin x}{1+\cos x}} \cdot \frac{1}{\frac{(1+\cos x)^2+\sin^2 x}{(1+\cos x)^2}} \cdot \frac{\cos x(1+\cos x) - \sin x(-\sin x)}{(1+\cos x)^2} =$$

$$= \frac{1}{\operatorname{arctg} \frac{\sin x}{1+\cos x}} \cdot \frac{\cos x + \overbrace{\cos^2 x + \sin^2 x}^{1}}{1 + 2\cos x + \underbrace{\cos^2 x + \sin^2 x}_{1}} =$$

$$= \frac{1}{\operatorname{arctg} \frac{\sin x}{1+\cos x}} \cdot \frac{\cancel{\cos x + 1}}{\underbrace{2 + 2\cos x}_{2(1+\cancel{\cos x})}} = \boxed{\frac{1}{2 \operatorname{arctg} \frac{\sin x}{1+\cos x}}}$$

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⑤ $y = \dfrac{1}{\log(3x^2+x+1)} = (\log(3x^2+x+1))^{-1}$

$$y' = (-1) \cdot \left(\log\left(3x^2+x+1\right)\right)^{-2} \cdot \left(\log\left(3x^2+x+1\right)\right)' =$$

$$= \frac{-1}{\log^2(3x^2+x+1)} \cdot \frac{1}{(3x^2+x+1) \cdot \ln 10} \cdot \left(3x^2+x+1\right)' =$$

$$= \boxed{\frac{-(6x+1)}{(3x^2+x+1) \cdot \ln 10 \cdot \log^2(3x^2+x+1)}}$$

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⑥ $y = 5^{x^2-2x+1}$

$$y' = 5^{x^2-2x+1} \cdot \ln 5 \cdot \left(x^2-2x+1\right)' = \boxed{(2x-2) \ln 5 \cdot 5^{x^2-2x+1}}$$

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⑦ $y = \dfrac{\cos x}{2\sin^2 x}$

$$y' = \frac{(\cos x)' \cdot 2\sin^2 x - \cos x \left(2\sin^2 x\right)'}{\left(2\sin^2 x\right)^2} = \frac{-\sin x \cdot 2\sin^2 x - \cos x \cdot 2 \cdot 2(\sin x)^{2-1} \cdot (\sin x)'}{4\sin^4 x} =$$

$$= \frac{-2\sin^3 x - 4\cos x \cdot \sin x \cdot (\cos x)}{4\sin^4 x} = \frac{-2\sin^3 x - 4\cos^2 x \sin x}{4\sin^4 x} =$$

$$= \frac{-\cancel{2}\cancel{\sin x}\overbrace{(\sin^2 x + \cos^2 x}^{1} + \cos^2 x)}{\cancel{4}_{2}\cancel{\sin^4}^{3} x} = \boxed{-\frac{1+\cos^2 x}{2\sin^3 x}}$$

⑧ $y = \dfrac{2x^3}{\sqrt{x}} + \dfrac{x}{2\sqrt[3]{x}} = 2\dfrac{x^3}{x^{\frac{1}{2}}} + \dfrac{1}{2}\dfrac{x}{x^{\frac{1}{3}}} = 2x^{\frac{5}{2}} + \dfrac{1}{2}x^{\frac{2}{3}}$ (jen úpravy mocnin, pak až derivovat)

$$y' = 2 \cdot \frac{5}{2}x^{\frac{5}{2}-1} + \frac{1}{2} \cdot \frac{2}{3}x^{\frac{2}{3}-1} = 5x^{\frac{3}{2}} + \frac{1}{3}x^{-\frac{1}{3}} = \boxed{5\sqrt{x^3} + \frac{1}{3\sqrt[3]{x}}}$$

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⑨ $y = \dfrac{x}{\sqrt{1-x^2}} \cdot \arcsin x$

$$y' = \left(\frac{x}{\sqrt{1-x^2}}\right)' \cdot \arcsin x + \frac{x}{\sqrt{1-x^2}} \cdot (\arcsin x)' =$$

$$= \frac{(x)' \cdot \sqrt{1-x^2} - x \cdot \left(\sqrt{1-x^2}\right)'}{\left(\sqrt{1-x^2}\right)^2} \cdot \arcsin x + \frac{x}{\sqrt{1-x^2}} \cdot \frac{1}{\sqrt{1-x^2}} =$$

$$= \frac{1 \cdot \sqrt{1-x^2} - x \cdot \frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}} \cdot \left(1-x^2\right)'}{1-x^2} \cdot \arcsin x + \frac{x}{1-x^2} =$$

$$= \frac{\sqrt{1-x^2} - \frac{x}{2\sqrt{1-x^2}} \cdot (-2x)}{1-x^2} \cdot \arcsin x + \frac{x}{1-x^2} = \frac{\sqrt{1-x^2} + \frac{2x^2}{2\sqrt{1-x^2}}}{1-x^2} \cdot \arcsin x + \frac{x}{1-x^2} =$$

$$= \frac{\frac{1-\cancel{x^2}+\cancel{x^2}}{\sqrt{1-x^2}}}{1-x^2} \cdot \arcsin x + \frac{x}{1-x^2} = \boxed{\frac{1}{\sqrt{1-x^2} \cdot (1-x^2)} \cdot \arcsin x + \frac{x}{1-x^2}}$$

lze ještě upravit takto:

$$\frac{\arcsin x + x \cdot \sqrt{1-x^2}}{\sqrt{1-x^2} \cdot (1-x^2)} = \boxed{\frac{\arcsin x + x \cdot \sqrt{1-x^2}}{\sqrt{(1-x^2)^3}}}$$

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⑩ $y = \operatorname{tg} \sqrt{x} + \underbrace{\operatorname{tg} \frac{\pi}{4}}_{\text{konstanta}}$

$$y' = \frac{1}{\cos^2 \sqrt{x}} \cdot (\underbrace{\sqrt{x}}_{x^{\frac{1}{2}}})' + 0 = \frac{1}{\cos^2 \sqrt{x}} \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} = \boxed{\frac{1}{2\sqrt{x} \cos^2 \sqrt{x}}}$$