Kořeny, rozklad, znaménko polynomu

a) $3x^3 - 5x^2 - 4x + 4$ (možné kořeny: $\pm 1, \pm 2, \pm 4$)

	3	−5	−4	4
2	3	1	−2	0

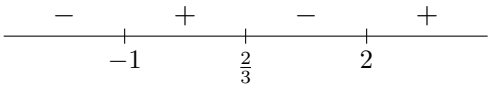
$\rightarrow (x-2)$

$3x^2 + x - 2 = 3(x+1)\left(x - \frac{2}{3}\right)$

$$D = 1^2 - 4 \cdot 3 \cdot (-2) = 1 + 24 = 25$$

$$x_{1,2} = \frac{-1 \pm 5}{3 \cdot 2} = \begin{cases} -\frac{6}{6} = -1 \\ \frac{4}{6} = \frac{2}{3} \end{cases}$$

rozklad: $3(x-2)(x+1)\left(x - \frac{2}{3}\right)$
kořeny: $x_1 = 2$, $x_2 = -1$, $x_3 = \frac{2}{3}$
znaménko:



---

b) $3x^3 - 8x^2 + 7x - 2$ (možné kořeny: $\pm 1, \pm 2$)

	3	−8	7	−2
1	3	−5	2	0

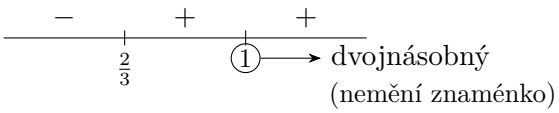
$\rightarrow (x-1)$

$3x^2 - 5x + 2 = 3(x-1)\left(x - \frac{2}{3}\right)$

$$D = (-5)^2 - 4 \cdot 3 \cdot 2 = 25 - 24 = 1$$

$$x_{1,2} = \frac{5 \pm 1}{2 \cdot 3} = \begin{cases} \frac{6}{6} = 1 \\ \frac{4}{6} = \frac{2}{3} \end{cases}$$

kořeny: $x_1 = 1$, $x_2 = 1$, $x_3 = \frac{2}{3}$
rozklad: $3(x-1)^2\left(x - \frac{2}{3}\right)$
znaménko:



dvojnásobný
(nemění znaménko)

c) $x^4 + 3x^3 - 7x^2 - 9x + 12$ (možné kořeny: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$)

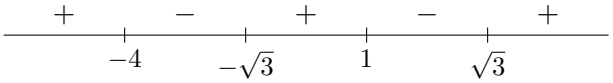
	1	3	-7	-9	12	
1	1	4	-3	-12	0	$\rightarrow\ (x-1)$
-4	1	0	-3	0		$\rightarrow\ (x+4)$

$$x^2 + 0x - 3 = 0$$

$$x^2 - 3 = 0$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$

kořeny: $x_1 = 1$, $x_2 = -4$, $x_3 = \sqrt{3}$, $x_4 = -\sqrt{3}$
rozklad: $(x-1)(x+4)\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)$

znaménko:



---

d) $x^4 - 2x^3 + 2x - 1$ (možné kořeny: ±1)

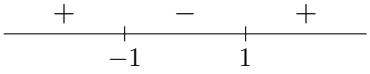
	1	-2	0	2	-1	
1	1	-1	-1	1	0	$\rightarrow\ (x-1)$
1	1	0	-1	0		$\rightarrow\ (x-1)$

$$x^2 + 0x - 1 = 0$$

$$(x-1)(x+1) = 0$$

kořeny: $x_{1,2,3} = 1$ (trojnásobný), $x_4 = -1$
rozklad: $(x-1)^3(x+1)$

znaménko:



e) $2x^3 - 3x^2 - 3x + 2$ (možné kořeny: $\pm 1, \pm 2$)

	2	−3	−3	2	
−1	2	−5	2	0	$\rightarrow\ (x+1)$

$2x^2 \quad -5x \quad +2$

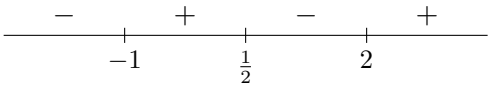
$D = (-5)^2 - 4 \cdot 2 \cdot 2 = 9$

$$x_{1,2} = \frac{5 \pm 3}{2 \cdot 2} = \begin{cases} \frac{8}{4} = 2 \\ \frac{2}{4} = \frac{1}{2} \end{cases}$$

kořeny: $x_1 = -1$, $x_2 = 2$, $x_3 = \frac{1}{2}$

rozklad: $2(x+1)(x-2)\left(x - \frac{1}{2}\right)$

znaménko:



---

f) $2x^4 - x^3 + 2x^2 - 3x = x\left(2x^3 - x^2 + 2x - 3\right)$ (možné kořeny: $\pm 1, \pm 3$)

$x_1 = 0$

$x_2 = 1$ ←

	2	−1	2	−3
1	2	1	3	0

$2x^2 \quad +x \quad +3$

$D = 1^2 - 4 \cdot 2 \cdot 3 < 0$ – nemá reálné kořeny

kořeny: $x_1 = 0$, $x_2 = 1$

rozklad: $x(x-1)\left(2x^2 + x + 3\right)$

znaménko:

