Pro funkci $f(x) = \operatorname{arctg} \frac{x-1}{x-2}$ určete:

a) $D(f)$ a vodorovnou asymptotu
b) tečna a normála v $x_0 = 1$

**a)**
$D(f) = \mathbb{R} - \{2\}$

$$\lim_{x\to\infty} \operatorname{arctg} \frac{x-1}{x-2} = \operatorname{arctg} 1 = \frac{\pi}{4}$$
$$\lim_{x\to-\infty} \operatorname{arctg} \frac{x-1}{x-2} = \operatorname{arctg} 1 = \frac{\pi}{4}$$

vodorovná asymptota $y = \frac{\pi}{4}$ pro $x \to \pm\infty$

**b)**

$$f'(x) = \frac{1}{1 + \left(\frac{x-1}{x-2}\right)^2} \cdot \frac{1\cdot(x-2) - (x-1)\cdot 1}{(x-2)^2} = \frac{1}{\frac{x^2-4x+4+x^2-2x+1}{(x-2)^2}} \cdot \frac{x-2-x+1}{(x-2)^2} =$$
$$= \frac{-1}{2x^2 - 6x + 5}$$

$$f'(1) = \frac{-1}{2-6+5} = \frac{-1}{1} = -1$$

$t\colon\ y - y_0 = f'(x_0)\cdot(x - x_0)$ $\qquad$ $t\colon\ y - 0 = -1\cdot(x-1)$

$$\underline{y = 1 - x}$$

$n\colon\ y - y_0 = -\frac{1}{f'(x_0)}\cdot(x - x_0)$ $\quad$ $n\colon\ y - 0 = 1\cdot(x-1)$

$$\underline{y = x - 1}$$

$f(1) = \operatorname{arctg} 0 = 0$

Pro funkci $f(x) = \arcsin(x^2)$ určete:

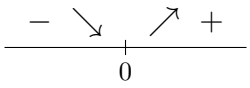
a) $D(f)$, intervaly monotonie a lokální extrémy
b) rovnici tečny a normály v $x_0 = \frac{1}{\sqrt{2}}$

**a)**

$D(f) = \langle -1, 1 \rangle$

$$f'(x) = \frac{1}{\sqrt{1-x^4}} \cdot 2x$$

$f'(x) = 0$ pro $x = 0$




rostoucí $\langle 0, 1 \rangle$
klesající $\langle -1, 0 \rangle$

$f(0) = \arcsin 0 = 0$ lokální minimum $[0, 0]$
$f(1) = \arcsin 1 = \sin \frac{\pi}{2} = 1$ lokální maximum $\left[1, \frac{\pi}{2}\right]$, $\left[-1, \frac{\pi}{2}\right]$

**b)**

$$f'\left(\frac{1}{\sqrt{2}}\right) = \frac{2 \cdot \frac{1}{\sqrt{2}}}{\sqrt{1-\frac{1}{4}}} = \frac{\sqrt{2}}{\sqrt{\frac{3}{4}}} = \frac{\sqrt{2}}{\frac{\sqrt{3}}{2}} = \frac{2 \cdot \sqrt{2}}{\sqrt{3}}$$

$$y_0 = f(x_0) = \arcsin \frac{1}{2} = \frac{\pi}{6}$$

$t\colon\ y - y_0 = f'(x_0) \cdot (x - x_0)$ $\qquad t\colon\ y - \frac{\pi}{6} = \frac{2 \cdot \sqrt{2}}{\sqrt{3}} \cdot \left(x - \frac{1}{\sqrt{2}}\right)$

$n\colon\ y - y_0 = -\frac{1}{f'(x_0)} \cdot (x - x_0)$ $\quad n\colon\ y - \frac{\pi}{6} = -\frac{\sqrt{3}}{2 \cdot \sqrt{2}} \cdot \left(x - \frac{1}{\sqrt{2}}\right)$

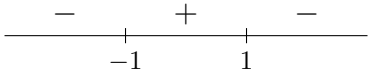
Pro funkci $f(x) = \ln \frac{x+1}{1-x}$ určete:

a) $D(f)$
b) lokální extrémy a intervaly monotonie
c) inflexní body a intervaly konvexnosti

**a)**

$\frac{x+1}{1-x} > 0$

$D(f) = (-1, 1)$




**b)**

$$f'(x) = \frac{1}{\frac{x+1}{1-x}} \cdot \frac{1 \cdot (1-x) - (x+1) \cdot (-1)}{(1-x)^2} = \frac{1-x}{x+1} \cdot \frac{1-x+x+1}{(1-x)^2} = \frac{2}{(x+1) \cdot (1-x)} = \frac{2}{1-x^2} =$$
$$= 2 \cdot \left(1 - x^2\right)^{-1}$$

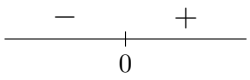
$\Rightarrow$ nemá extrémy, všude rostoucí, $f'(x) > 0$ v $(-1, 1)$

**c)**

$$f''(x) = -2 \cdot \left(1 - x^2\right)^{-2} \cdot (-2x) = \frac{4x}{\left(1 - x^2\right)^2}$$

$f''(x) = 0$ pro $x = 0$

konvexní v $(0, 1)$
konkávní v $(-1, 0)$
inflexní bod $I\,[0, 0]$



Pro funkci $f(x) = e^{\frac{1}{x}}$ určete:

a) inflexní body, intervaly konvexnosti a konkávnosti
b) vodorovnou asymptotu

**a)**

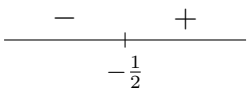
$D(f) = \mathbb{R} - \{0\}$

$$f'(x) = e^{\frac{1}{x}} \cdot \left(\frac{1}{x}\right)' = -1 \cdot x^{-2} \cdot e^{\frac{1}{x}} = -\frac{1}{x^2} \cdot e^{\frac{1}{x}}$$

$$f''(x) = -1 \cdot (-2) \cdot x^{-3} \cdot e^{\frac{1}{x}} + \left(-1 \cdot x^{-2} \cdot e^{\frac{1}{x}} \cdot (-1) \cdot x^{-2}\right) = \frac{2 \cdot e^{\frac{1}{x}}}{x^3} + \frac{e^{\frac{1}{x}}}{x^4} = \frac{(2x+1) \cdot e^{\frac{1}{x}}}{x^4}$$

$f''(x) = 0$ pro $2x + 1 = 0$, tj. $x = -\frac{1}{2}$

konvexní $\left(-\frac{1}{2}, 0\right) \cup (0, \infty)$
konkávní $\left(-\infty, -\frac{1}{2}\right)$
inflexní bod $I\left[-\frac{1}{2}, e^{-2}\right]$




**b)**

$\lim_{x \to \infty} e^{\frac{1}{x}} = e^0 = 1$

$\lim_{x \to -\infty} e^{\frac{1}{x}} = e^0 = 1$

$\underline{y = 1}$ pro $x \to \pm\infty$

Pro funkci $f(x) = \frac{1}{e^x}$ určete:

a) inflexní body, intervaly konvexnosti a konkávnosti
b) vodorovnou asymptotu

**a)**
$D(f) = \mathbb{R}$

$$f'(x) = \frac{-e^x}{(e^x)^2} = -\frac{1}{e^x}$$
$$f''(x) = \frac{e^x}{(e^x)^2} = \frac{1}{e^x}$$

$\Rightarrow$ inflexní body nemá, všude konvexní

**b)**

$$\lim_{x\to\infty} \frac{1}{e^x} = 0$$
$$\lim_{x\to-\infty} \frac{1}{e^x} = \infty$$

vodorovná asymptota $y = 0$ pro $x \to \infty$

Je daná funkce $f(x) = \frac{e^{3x}}{x^2}$, určete:

a) $D(f)$, lokální extrémy a intervaly monotonie
b) inflexní body a intervaly konvexnosti

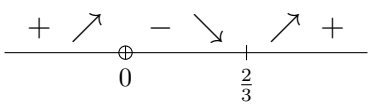
$D(f) = \mathbb{R} - \{0\}$

**a)**

$$f'(x) = \frac{(e^{3x})' \cdot x^2 - e^{3x} \cdot 2x}{x^4} = \frac{3 \cdot e^{3x} \cdot x^2 - 2x \cdot e^{3x}}{x^4} = \frac{e^{3x} \cdot x \cdot (3x-2)}{x^4} = \frac{e^{3x} \cdot (3x-2)}{x^3}$$

$f'(x) = 0$ pro $3x - 2 = 0$, tj. $x = \frac{2}{3}$

rostoucí v $(-\infty, 0)$ a $\left(\frac{2}{3}, \infty\right)$
klesající v $\left(0, \frac{2}{3}\right)$




$$f\left(\tfrac{2}{3}\right) = \frac{e^{3 \cdot \frac{2}{3}}}{\left(\frac{2}{3}\right)^2} = \frac{9}{4} \cdot e^2 \qquad \text{lokální minimum } E\left[\tfrac{2}{3},\ \tfrac{9}{4} \cdot e^2\right]$$

**b)**

$$f''(x) = \frac{(e^{3x} \cdot (3x-2))' \cdot x^3 - (e^{3x} \cdot (3x-2) \cdot 3x^2)}{x^6} =$$
$$= \frac{(e^{3x} \cdot 3 \cdot (3x-2) + e^{3x} \cdot 3) \cdot x^3 - (e^{3x} \cdot (9x^3 - 6x^2))}{x^6} =$$
$$= \frac{e^{3x} \cdot x^2 \cdot (9x^2 - 6x + 3x - 9x + 6)}{x^6} = \frac{e^{3x} \cdot (9x^2 - 12x + 6)}{x^4} = \frac{3 \cdot e^{3x} \cdot (3x^2 - 4x + 2)}{x^4}$$

$$f''(x) = 0 \text{ pro } 3x^2 - 4x + 2 = 0$$
$$D = b^2 - 4 \cdot a \cdot c = 16 - 4 \cdot 3 \cdot 2 = -8 < 0$$

$\Rightarrow$ nemá inflexní body, všude $f''(x) > 0$, všude konvexní $(-\infty, 0) \cup (0, \infty)$